# PLANE EXPLOSION WAVE IN SOILS

R. G. Yakupov

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It is shown in the article that simple analytical expressions can be used to determine the velocity of the front of a plane shock wave and the parameters of the soil behind the front in the region from the boundary of the charge of explosive up to the moment of the emission of sound. The calculated data are compared with the results of field tests. The calculations are based on the use of a diagram of the compression of the soil constructed taking account of the results of the experiment.

1. A number of experimental and theoretical publications [1-5], in which the soil is modeled as a "plastic gas," an incompressible liquid, and an elasticoplastic body obeying the laws of the theory of plasticity, have been devoted to the problem under consideration.

In the present work a model of the soil proposed in [6] is adopted. It is assumed that the flow starts immediately behind the front and that the density in a particle after the passage of the shock wave remains unchanged.

For description of the medium we have two conditions: the condition of plasticity and the law of volumetric compression

$$J_2 = (kp + b)^2/6 \tag{1.1}$$

$$p(\theta) = p^0\theta^n \tag{1.2}$$

where $J_2$ is the second invariant of the deviator of the stress tensor; p is the mean hydrostatic pressure; k and b are coefficients characterizing, respectively, the internal friction and the adhesion of the soil; $\theta = 1-\rho_0/\rho$ is the volumetric deformation; $\rho_0$ is the initial density; $\rho$ is the density of the particles behind the front of the shock wave.

From relationships (1.1) and (1.2) we can obtain the law of compression for a monoaxial deformed state ($\varepsilon_y = \varepsilon_z = 0$, $\varepsilon_x = \theta$):

$$\sigma_x = (1 + \sqrt{2}k/3)\, p^0\varepsilon_x{}^n + \sqrt{2}b/3 \tag{1.3}$$

The wave is propagated in the direction of the x axis and, in what follows, we shall assume that $\sigma_x = \sigma$, $\varepsilon_x = \varepsilon$.

Let us determine the parameters of a shock wave, propagating in a non-water-saturated sandy soil of fractured structure, having a moisture content of w=15-17%. The compressibility of the soil varies; the coefficients k=1.25, b= 0. The compression diagrams (1.2) used in the calculation are given on Fig. 1. Curve 1 illustrates the compression diagram of a soil with a volumetric weight of its skeleton $\gamma_1 = (1.225\text{-}1.295)\cdot 10^4$ N/m$^3$, curve 2 a soil with a volumetric weight of its skeleton $\gamma_2 = (1.32\text{-}1.37)\cdot 10^4$ N/m$^3$. The parts of the curves corresponding to the experiment of [7] are shown by the solid line and are described by an exponential law (1.2). The power exponent of the compression n and the parameter $p^0$, respectively, for curves 1 and 2, are equal to 2.5, 3.0, and $14.8\cdot 10^8$, $3.64\cdot 10^8$ N/m$^2$. The measurements of [7, 8] show that, with high pressures, the densification attains a limiting value and then remains constant. Therefore, the compression diagram must have an asymptote corresponding to the pressure, and tending to infinity. We assume that the limiting values of the deformations for curve 1 fluctuate within the limits $\varepsilon_* = 0.085\text{-}0.1$;

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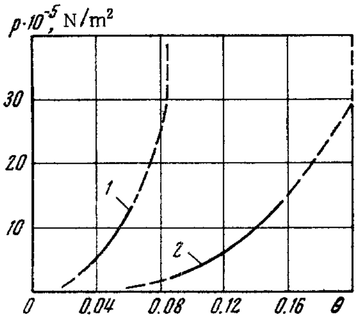


Fig. 1

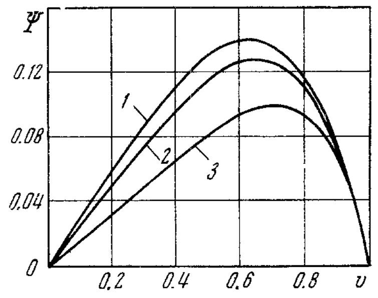


Fig. 2

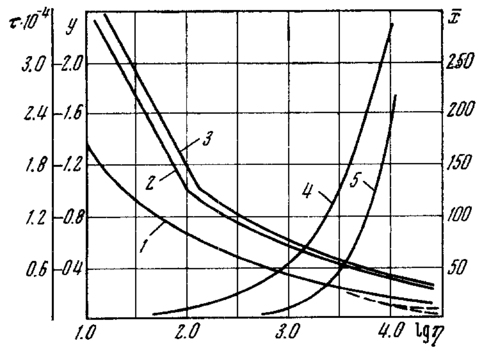


Fig. 3

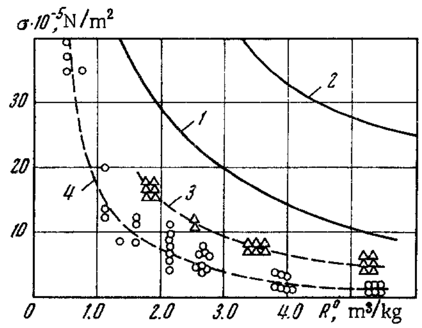


Fig. 4

for curve 2 $\varepsilon_* = 0.2$. The latter are taken from an analysis of the experimental data of [5] for non-water-saturated soils.

In a homogeneous soil, occupying all the space between the planes $x_0 = \pm a$, there is located a charge of explosive which, with an explosion, goes over instantaneously into a high-pressure gas without change of volume. The initial pressure in the products of an instantaneous detonation

$$p_1 = \rho_c D^2 / 2\,(1 + \lambda)$$

where D is the detonation rate; $\rho_c$ is the density of the charge of explosive; $\lambda$ is the isentropic index for the detonation products. Taking $D = 7 \cdot 10^3$ m/sec, $\rho_c = 1.6$ kg/m³, $\lambda = 3$, we find $p_1 = 10^{10}$ N/m².

Using the relationship of the equality of the displacements at the boundary of the cavity at the moment when the detonation products flow out into the medium [9]

$$\frac{2D}{\lambda - 1} \sqrt{\frac{\lambda}{2(\lambda + 1)}} \left[1 - \left(\frac{p_2}{p_1}\right)^{(\lambda-1)/2\lambda}\right] = \sqrt{\frac{p_2 \varepsilon_*}{\rho_0}}$$

expression (1.3), and the condition of the equality of the stresses at the boundary between the limiting and variable densifications, we find the initial pressure in the soil $p_2 \approx 2 \cdot 10^9$ N/m². The application of a given dynamic load to the surface $x_0 = \pm a$ brings about the formation of a shock wave in the soil, which is then propagated over the medium.

By virtue of symmetry, we consider the process from one side, from the middle of the charge. The origin of coordinates is at the point of symmetry.

The motion of the soil after the passage of the shock wave in Lagrange variables is described by the equations

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0} \frac{\partial \sigma}{\partial x_0}, \qquad u = \frac{\partial x}{\partial t}, \qquad \frac{\partial x}{\partial x_0} = 1 - \varepsilon \tag{1.4}$$

where $x_0$ and $x$ are the coordinates of a particle in accordance with Lagrange and Euler; $t$ is the time; $u$ is the velocity.

The boundary conditions for (1.4) are the expressions of the laws of conservation at the front of the wave, the condition of the continuity of the instantaneous coordinate, and the equality of the pressures at the boundary of the expanding cavity,

$$\sigma_f = \rho_0 \varepsilon_f (dx_f/dt)^2, \quad u_f = \varepsilon_f (dx_f/dt), \quad x_f = x_0, \quad p_\varkappa = p_2 (a/x)^\gamma \tag{1.5}$$

The subscript $f$ means that the values relate to the front of the wave; $p_\varkappa$ is the pressure at the boundary of the cavity. As the polytropic index we take $\gamma = 1.25$.

We write the solution of (1.4) taking account of (1.5) [2]

$$\bar{x} = s + \int_s^{\bar{x}_f} \varepsilon_f [y(s)]\, ds, \qquad \bar{\sigma} = (\bar{x}_f - s)\, \varphi(y)\, (dy / d\bar{x}_f) + \varepsilon_f y^2$$

$$\bar{x} = \frac{x}{a}, \quad s = \frac{x_0}{a}, \quad \bar{x}_f = \frac{x_f}{a}, \quad \bar{\sigma} = \frac{\sigma}{p_2}, \quad \tau = \frac{t}{a}\left(\frac{p_2}{\rho_0}\right)^{1/2}, \quad y = \frac{dx_f}{d\tau} \tag{1.6}$$

$$\varphi(y) = \frac{y\, d(\varepsilon_f y)}{dy}$$

where $y$ is the dimensionless velocity of the front of the wave; the $\varepsilon_f(y)$ is known from the condition at the front of the wave and (1.3), and has the form

$$\varepsilon_f = \beta y^m, \ \beta = [(1 + \sqrt{2}k/3)\ (p^0/p_2)]^{-m/2}, \ m = 2/(n-1) \tag{1.7}$$

Substitution of relationships (1.6) into the last expression of (1.5) followed by differentiation gives an equation for the velocity of the front of the wave:

$$\frac{d}{d\eta}\left[\varphi(y)\,\eta\,\frac{dy}{d\eta} + \varepsilon(y)\ y^2\right]^{-1/\gamma} = \varepsilon(y), \quad y^2(0)\,\varepsilon[y(0)] = 1, \quad \eta = \bar{x}_f - 1 \tag{1.8}$$

After replacement of the variables

$$v = A_0^{-1} y \eta^{-\omega}, \ d\ln\eta/dv = 1/\Psi(v) \tag{1.9}$$

(1.8) assumes the form [2]

$$\frac{d\Psi}{dv} - \frac{v}{\Psi}\frac{m+2}{m+1}\omega\mu^{-1/\gamma}\left[\mu + (m+1)\frac{\Psi}{v}\right]^{\alpha} v^{-1/\omega} + \frac{m+2}{m+1}\omega\mu\frac{v}{\Psi} + (m+1)\frac{\Psi}{v} + \frac{m+2}{m+1}\mu + (m+2)\,\omega = 0$$

$$A_0^{-1/\omega} = -\beta^{-\alpha}\frac{m+2}{\gamma}\omega\mu^{-1/\gamma}, \quad \alpha = 1 + \frac{1}{\gamma}, \quad \omega = -\frac{\gamma}{2 + m + m\gamma} \tag{1.10}$$

$$\mu = \omega(m+1) + 1$$

The boundary conditions for (1.10) are

$$\Psi = 0, \ d\Psi/dv = -\omega\ (v = 0), \ \Psi = 0\ (v = 1)$$

2. We solve Eq. (1.8) for the region of constant densification ($m = 0$, $\varepsilon_* = \beta$) [2]:

$$y = \frac{1}{\beta\eta}\left\{\frac{2}{\beta(1-\gamma)(2-\gamma)}\left[1 + \beta\eta(2-\gamma)(1+\beta\eta)^{1-\gamma} - (1+\beta\eta)^{2-\gamma}\right]\right\}^{1/2} \tag{2.1}$$

The dimensionless stresses, with constant and variable densification at the front are determined, respectively, by the expressions

$$\bar{\sigma}_f = \beta y^2, \qquad \bar{\sigma}_f = \beta y^{m+2}$$

From the condition of the equality of the stresses at the moment of the transition from a constant densification to a variable densification, it follows that solution (2.1) is valid in the interval $y(0) \geq y \geq 1$.

To determine the motion of the front of the wave in the region of variable densification, we integrate Eq. (1.10) numerically in a digital computer. The curve of the dependence $\Psi(v)$ is shown on Fig. 2. Curves 1, 2, and 3 on Fig. 2 correspond to the values $n = 3$, 2.5, and 1.8. Using the curves of $\Psi(v)$ obtained, we approximate the right-hand part of the second relationship of (1.9) by the expression

$$1/\Psi = (\xi - v)/\varkappa v(1 - v) \tag{2.3}$$

and, after integration, we find

$$\eta = C\,[v^{\xi}/(1 - v)^{\xi - 1}]^{1/\varkappa} \tag{2.4}$$

where $\xi$ and $\varkappa$ are parameters. We determine the integration constant and the initial value of $v$ in (2.4) from the condition of the conjugation of $y(\eta)$ with solution (2.1).

It can be shown by direct calculations that the values of the dimensionless velocities of the front of the wave determined using relationships (2.4), (1.9), and from the formula

$$y = A_0\eta^{\omega} \tag{2.5}$$

coincide. Expression (2.5) is an asymptotic solution of (1.8). Therefore, in the region of variable densification, formula (2.5) can be used. The coefficient $A_0$ is determined in this case from the condition of conjugation and differs from the previous value only insignificantly.

Curves of the dependence $y(\eta)$, plotted using formulas (2.1), (2.5) and experimental [5] curves are shown on Fig. 3. Curves 1, 2, and 3 relate to a sandy soil with the compression power exponents $n = 3$, 2.5, and 3, and a limiting deformation $\varepsilon_* = 0.2$, 0.1, and 0.085. Experimental values of the rate of propagation of the wave were determined with a value of the moisture content $w = 10$–12%, and are shown by the dotted curve on Fig. 3.

With a known velocity of the wave, all the parameters of the front of the wave and the motion of the soil can be determined. For further calculations, it is convenient to approximate (2.1) by the expression

$$y = (a\eta^b + c)^{-1}$$

Curves 4 and 5 on Fig. 3 show the change in the boundary of the cavity $\overline{x}(\eta)$ and the time of the propagation of the shock wave $\tau(\eta)$ with a limiting deformation $\varepsilon_* = 0.085$. Fig. 4 gives calculated curves of the change in the dependence of the stresses at the front on the relative distance $\sigma(R^0)$ (curves 1 and 2) and experimental data from measurement of the stresses in field tests [5] (curves 3 and 4 correspond to sands with w = 10-12% and curve 2 to 4%). In plotting the stress curves 1 and 2, values of the velocity of the wave given, respectively, in the form of curves 1 and 2 on Fig. 3, were used. The coordinate $\eta$ and the relative distance $R^0$ are connected by the dependence

$$\eta = 2 \cdot 10^3 \rho_c R^0$$

The zone of propagation of the shock wave and the parameters of the soil behind the front depend considerably on the compressibility of the soil. Therefore, for a more exact qualitative analysis, data on the compressibility of the soil with high values of the pressure are required.

Approximation of the compression diagram by the curve given on Fig. 1 permits carrying out the calculations using the simple formulas (2.1) and (2.5).

The calculated and experimental data are in satisfactory agreement.

The solution (2.5) is valid up to the moment of the emission of sound, after which the region of the propagation of elasticoplastic deformation begins.

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